FYUGP/3rd Sem/25(NEP)New

2025

Four Year Under Graduate Programme (FYUGP)

3rd Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-301

(Abstract Algebra)

Full Marks : 50

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2\times5=10$

(a) Find all units in the ring

$$\mathbb{Z}[i]=\{a+bi\in\mathbb{C}\,|\,a,b\in\mathbb{Z}\}$$

(b) How many group homomorphisms are there from $\mathbb{Z}_3$ to $\mathbb{Z}_2$?

(c) Let G be a group and $a\in G$. If $o(a)=17$ then find $o(a^3)$.

P.T.O.

(d) Show that every element of $(\mathbb{Q}/\mathbb{Z},+)$ is of finite order whereas $(\mathbb{Q}/\mathbb{Z},+)$ is of infinite order.

(e) Find the order of $(1234)\circ(567)$ in S_7.

(f) Let G be a cyclic group of order 30. Find the number of elements of order 6 in G.

(g) Let G and G' be two groups, and Let $\phi : G \to G'$ be a homomorphism. Prove that ker φ is a normal subgroup of G.

(h) Give an example of a ring S with identity whose all non-trivial subrings do not contain the identity element. What is the intersection of all subfields of $\mathbb{R}$? 1+1

2. (a) Let H be a subgroup of a group G Show that for any $g \in G$, the set $K = \left\{ghg^{-1} \middle| h \in H\right\}$ is a subgroup of G. Also, show that $|K| = |H|$.

(b) Prove that $(\mathbb{Q},+)$ is a non-cyclic group.

(c) Find the maximum possible order of an element in S_9.

(d) How many generators are there in a group of order 23. 5+2+2+1

3. (a) Let G be a group and H be a subgroup of G such that $[G : H] = 2$. Prove that $x^2 \in H$ for all $x \in G$.

(b) List all normal subgroups of S_4.

(c) Prove that every group is isomorphic to some subgroup of the group $A(S)$ of all permutations on some set S. 3+2+5

4. (a) Is the converse of Lagrange's theorem true? Justify your answer.

(b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 7 & 5 & 9 & 6 & 8 & 3 & 4 \end{pmatrix}$ as a product of transpositions. Is it even or odd?

(c) Let G be a finite group of order n, and m be a positive integer such that $gcd(m, n) = 1$. Show that the mapping $\varphi : G \to G$ defined by $\varphi(x) = x^m$ is an isomorphism. 3+2+5

5. (a) Find all group homomorphisms from $\mathbb{Z}_8$ to $\mathbb{Z}_{12}$.

(b) Find all idempotent elements in the ring $\mathbb{Z}_6 \times \mathbb{Z}_{12}$.

(c) Show that $4\mathbb{Z}/12\mathbb{Z} \simeq \mathbb{Z}_3$. 5+2+3

6. (a) Show that $\mathbb{Q}\left[\sqrt{2}\right] = \left\{a + b\sqrt{2} \in \mathbb{R} \mid a, b \in \mathbb{Q}\right\}$ is a subfield of $\mathbb{R}$.

(b) Prove that the ring $2\mathbb{Z}$ is not isomorphic to $5\mathbb{Z}$.

(c) Find all maximal ideals of $\mathbb{Z}_6$. 4+3+3

P.T.O.

7. (a) Let R be a commutative ring with identity. Prove that every maximal ideal of R is a prime ideal of R. Is the converse true? Justify your answer.

(b) Show that the set $I = \left\{a + b\sqrt{3} \in \mathbb{Z}[\sqrt{3}] \,\middle|\, a - b \text{ is an even integer}\right\}$ is an ideal in the ring $\mathbb{Z}[\sqrt{3}] = \left\{a + b\sqrt{3} \,|\, a, b \in \mathbb{Z}\right\}$.

(c) Prove that every finite integral domain is a field.

(d) Give an example of a Boolean ring. 3+3+3+1

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MATHEMATICS (Major)

Paper Code : MTMMJ DC-302

(Differential Equation)

Full Marks : 50

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2×5=10

(a) If the differential equation

$$\left(1+x^2y^3+\alpha x^2y^2\right)dx+\left(2+x^3y^2+x^3y\right)dy=0$$ is

exact, find the value of α.

(b) Find the orthogonal trajectories of the family of curves $r=a\theta$.

(c) Show that the initial value problem $\dfrac{dy}{dx}=y^{1/3}, y(0)=0$ has no unique solution.

P.T.O.

(d) Find the general solution of $(y-px)(p-1)=p$, where $p \equiv \frac{dy}{dx}$.

(e) Show that $x = 0$ is an ordinary point of $\left(x^2-1\right)y'' + xy' - y = 0$, but $x = 1$ is a regular singular point.

(f) Find a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}\sin x.$$

(g) Form a partial differential equation by eliminating the arbitrary function φ from

$$lx + my + nz = \varphi\left(x^2 + y^2 + z^2\right).$$

(h) Examine the nature of critical points of the system :

$$\frac{dx}{dt} = -4x - y$$

$$\frac{dy}{dt} = x - 2y$$

2. (a) Reduce the equation $\sin y \frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$ to a linear equation and hence solve it.

(b) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ 5+5

3. (a) Apply the method of variation of parameters to solve the differential equation :

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$

(b) Solve the system of equations :

$$\frac{dx}{dt}+5x+y=e^t$$

$$\frac{dy}{dt}-x+3y=e^{2t}$$ 5+5

4. (a) Using the method of undetermined coefficients, solve the differential equation :

$$\left(D^2-2D+3\right)y=x^2+\sin x,\ \text{where } D\equiv\frac{d}{dx}$$

(b) Reduce the equation $x^3p^2+x^2yp+a^3=0$ to Clairaut's form by the substitution $y = u$ and $x=\frac{1}{v}$, and obtain the complete primitive. 5+5

5. (a) Find the power series solution of the differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$ about $x = 0$.

P.T.O.

(b) Prove that $\left(1-2zt+t^2\right)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} P_n(z)t^n, |t|<1.$

5+5

6. (a) Solve $(x^2+y^2+yz)p+(x^2+y^2-xz)q = z(x+y)$ by Lagrange's method, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$.

(b) Find a complete integral of $pxy+pq+qy=yz$ by Charpit's method.

5+5

7. (a) Find the eigenvalues and eigenfunctions of the differential equation $\frac{d^2y}{dx^2}+\lambda y=0$, satisfying the boundary conditions $y'(0)=0, y'(1)=0$.

(b) Prove that $J_2'(x)=\left(1-\frac{4}{x^2}\right)J_1(x)+\frac{2}{x}J_0(x)$, where $J_n(x)$ is the Bessel's function of first kind.

5+5